

**St Stephen's School – Carramar Campus****Year 11 Mathematics Specialist****Test 2 part B****Total Marks: 33****Time Allowed: 35 mins****Resource Rich****Name:** _____**Mark:** _____**Teacher:** _____**Parent Signature:** _____**INSTRUCTIONS**

Permitted equipment:

- Two calculators complying with Curriculum Council requirements
- Two A4 pages (both sides) of notes
- Stationery and drawing equipment
- Refer to the attached formula sheet

Question 1. [1+2+2+4=9 marks]

The population of bacteria growing on an agar plate is modelled by the equation

$$P = 100(1.02)^t$$

where t is time in minutes.

- a. What is the initial (i.e.
- $t = 0$
-) population?

100 ✓

S

- b. What is the predicted population after one hour?

$$100 \times 1.02^{60} = 328 \checkmark$$

S

- c. To the nearest hour, how long must the plate be incubated before the population reaches ten million?

$$\begin{aligned} 100 \times 1.02^t &= 10\,000\,000 \\ 1.02^t &= 100\,000 \\ t &= 581 \text{ min } \checkmark \\ &\approx 10 \text{ hours. } \checkmark \end{aligned}$$

S

- d. Using base 10 logarithms, rearrange
- $P = 100(1.02)^t$
- to give an expression for
- t
- in terms of
- P
- (i.e. make
- t
- the subject of the equation.)

$$\begin{aligned} 1.02^t &= \frac{P}{100} \checkmark \\ \log 1.02^t &= \log \frac{P}{100} \checkmark \\ t \log 1.02 &= \log P - \log 100 \\ &= \log(P) - 2 \checkmark \\ t &= \frac{\log(P) - 2}{\log 1.02} \checkmark \end{aligned}$$
$$\frac{\log \frac{P}{100}}{\log 1.02}$$

C

Question 2. [1+1+1=3 marks]

Evaluate to three decimal places:

a. $\log_{10} 7 = 0.845$ S

b. $\log_2 7 = 2.807$ S

c. $\log_3 2 \times \log_5 1 = 0.000$ S

Question 3. [1+1+1+2 = 5 marks]

A report on Slashdot on 14 May 2009 included this quote:

Firefox could surpass IE market share as early as January 2013 if Firefox continues to gain 5 percent every year, even as IE drops 5 percent each year

- a. Let m represent the 2009 market share held by Firefox. Write an expression in terms of m for the market share that will be held by Firefox in t years time.

$$F = m(1.05)^t$$
 ✓ S

- b. Assuming the Slashdot report is true, write an expression in terms of m for the 2013 market share held by IE.

$$M \times 1.05^4 = 1.22m$$
 ✓ S

- c. Write an expression in terms of m for the 2009 market share held by IE.

$$M \times 1.05^4 \times 0.95^{-4}$$

$$= m \times \frac{1.05^4}{0.95^4} = 1.49m$$
 ✓ C

- d. Find m given that the total 2009 market share of Firefox and IE is 0.91.

$$m + 1.49m = 0.91$$

$$m = \frac{0.91}{2.49}$$

$$= 0.37$$
 ✓

$$(0.3651)$$
 ✓ C

Question 4. [3 marks]

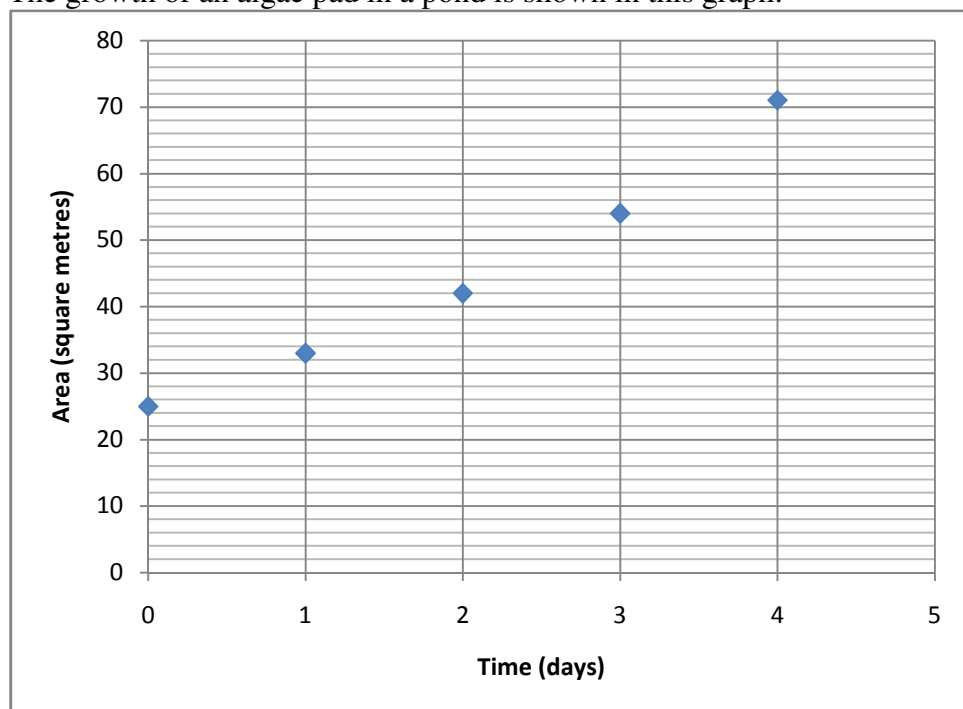
To a person on a boat moving with velocity $(3\mathbf{i} - 7\mathbf{j})$ km/h the wind appears to have velocity $(-\mathbf{i} + 3\mathbf{j})$ km/h. Determine the true velocity (in component form) of the wind.

$$\begin{aligned}\underline{V}_w - \underline{V}_b &= \underline{V}_w - \underline{V}_p \quad \checkmark \\ -\underline{i} + 3\underline{j} &= \underline{V}_w - (3\underline{i} - 7\underline{j}) \quad \checkmark \\ \underline{V}_w &= -\underline{i} + 3\underline{j} + 3\underline{i} - 7\underline{j} \quad \checkmark \\ &= (2\underline{i} - 4\underline{j}) \text{ km/h} \quad \checkmark\end{aligned}$$

C

Question 5. [4 marks]

The growth of an algae pad in a pond is shown in this graph:



k: 1 mark S
a: One ratio ✓
all ratios ✓
appropriate rounding ✓ S

If the growth is modelled using the equation $A = ka^t$, find the values of k and a .

$$k = 25 \text{ m}^2 \quad \checkmark$$

$$\frac{33}{25} = 1.32$$

$$\frac{42}{33} = 1.33 \quad \checkmark\checkmark$$

$$\frac{54}{42} = 1.29$$

$$\frac{71}{54} = 1.31$$

$$a \approx 1.3 \quad \checkmark$$

Question 6. [4 marks]Given $\mathbf{a} = (7\mathbf{i} - \mathbf{j})$ and $\mathbf{b} = (-11\mathbf{i} - 17\mathbf{j})$ find a vector in the same direction as \mathbf{b} and having the same magnitude as \mathbf{a} . Give your answer in component form rounded to one decimal place.

$$|\mathbf{a}| = \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2} \quad \checkmark$$

$$|\mathbf{b}| = \sqrt{11^2 + 17^2} = \sqrt{410} \quad \checkmark$$

$$\checkmark \frac{|\mathbf{a}|}{|\mathbf{b}|} \mathbf{b} = \frac{\sqrt{50}}{\sqrt{410}} (-11\mathbf{i} - 17\mathbf{j})$$

$$= (-3.8\mathbf{i} - 5.9\mathbf{j}) \quad \checkmark$$

S

Question 7. [3 marks]How far will a body moving with a constant velocity of $(5\mathbf{i} - 12\mathbf{j})\text{m/s}$ travel in one minute?

$$|5\mathbf{i} - 12\mathbf{j}| = \sqrt{5^2 + 12^2} \quad \checkmark$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13 \text{ m/s} \quad \checkmark$$

$$13 \times 60 = \underline{\underline{780 \text{ m}}} \quad \checkmark$$

S

Question 8. [1+1=2 marks]An aircraft is flying at 290km/h on a bearing of 220° a. What is the northerly component of its velocity? \checkmark

$$-290 \cos 40^\circ = -222 \text{ km/h} \quad \checkmark$$

$$(-222.1529)$$



S

b. What is the easterly component of its velocity?

$$-290 \sin 40^\circ = -186 \text{ km/h} \quad \checkmark$$

$$(-186.4084)$$

S

Formula Sheet

This formula sheet may be used for both parts of test 2.

Component vectors

If $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$

Then the magnitude of \mathbf{u} is given by

$$|\mathbf{u}| = \sqrt{a^2 + b^2}$$

and the direction of \mathbf{u} is given by

$$\tan \theta = \frac{b}{a}$$

Let \mathbf{u} be a vector of magnitude r and direction θ (measured anti-clockwise from the positive x -axis).

Then \mathbf{u} can be written in component form as

$$\mathbf{u} = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j}$$

Relative vectors

$${}_P\mathbf{r}_Q = \mathbf{r}_P - \mathbf{r}_Q$$

Index laws

$$a^n \times a^m = a^{n+m}$$

$$a^n \div a^m = a^{n-m}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$(a^n)^m = a^{n \times m}$$

$$(ab)^n = a^n \times b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^1 = a$$

$$a^0 = 1$$

Logarithms and Indices

$$\text{If } a^x = b \text{ then } \log_a b = x$$

Log laws

$$\log_a(bc) = \log_a b + \log_a c$$

$$\log_a \frac{b}{c} = \log_a b - \log_a c$$

$$\log_a(b^n) = n \log_a b$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$